All Saints' CofE Junior School Calculation Policy

Revised Sept 2014

Aims and Purposes

The purpose of the Calculations Policy is to develop a consistent approach to the way in which pupils record Mathematical calculations. Through this, a clear progression in their recording will become evident.

It aims to inform teaching staff, non-teaching staff, parents and governors as to how the school have chosen to record written calculations. It is to supplement the existing Mathematics policy and to support planning using the National Curriculum 2014.

This policy is designed to aid planning and to be used as a working document to ensure consistency throughout the school.

The Maths TLR will support all staff with their planning and understanding of this policy

Teaching and Learning

Children need to be able to understand and use one formal written method for each operation. They should be working towards a method that helps them to calculate efficiently and one that can be understood by other people.

Some children will not need to progress through all the methods within this policy

Written methods should complement mental methods, but children should be able to choose the most efficient method for a problem, be it mental, written or using a calculator (if possible).

Children should be taught written methods so that they can represent practical work and so they can record and explain mental calculations. It also helps them to keep track of procedures in longer problems and lay the foundations for calculations in algebra.

Children should be encouraged to approximate answers using mental methods for every operation before carrying out the formal written calculation. They should be able to explain their estimates.

The written strategies will be used for straightforward algorithms i.e. 68 + 127 =, but they will also be used when working with different units, fractions, decimals, percentages and word problems.

Using and Applying

There needs to be a large emphasis on using and applying in maths lessons. Also giving our children the chance to make their own decisions on what operations to use can inform APP judgements.

The formal written methods for the four operations should be taught before children are presented with problems that requires their application in a real life context. Problem solving should be introduced as part of the sequence of teaching the formal written method.

Each new method should be practised within a real life context including working with money and measure where appropriate.

Addition

To add successfully, children need to be able to:

- Relate addition to counting on
- Recognise that addition can be done in any order
- Use practical and informal written methods to support addition of a one digit number or a multiple of ten to a one digit or two digit number
- Use practical and informal methods to add two digit numbers
- Use efficient written methods to add whole numbers and decimals with up to 2 decimal places (including money)

Any calculations should be presented **horizontally** to support addition using number lines
As children get older they can adapt the calculation and change the layout appropriately to the needs of their method of working out

Explaining and modelling the commutative law should be carried out all the way throughout the school:

3 + 4 = 7 ⇨ 4 + 3 = 7

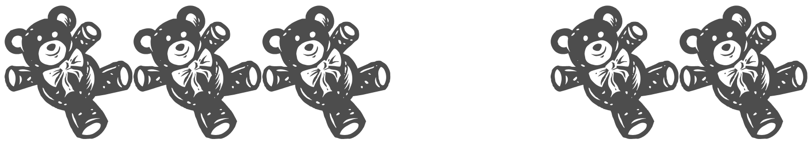
Stage 1: Practical Exploration of addition

Practical experience of finding '**1** more than', '**10** more than', 'What is **1** more than **3**?' **'3 + 1 = 4'**.

Relate addition to combining **2** groups of objects and then **3** groups of objects by counting all.

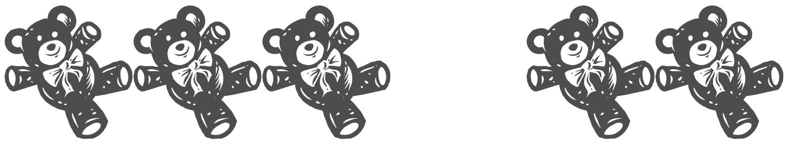
eg Count out **4** cakes, count out **3** cakes. How many altogether? Count all cakes.

Teacher models recording method, child then uses teacher format to record as appropriate.
E.g. Milly had 3 bears. She was given 2 more. How many does she have now?



"3 and 2 makes 5 altogether"

Pictorial recording alongside numbers (such as number cards, computer record etc.)



3	2

When children are deemed ready they can move on to recording of number sentences after modelling by the teacher. At this stage children should be encouraged to think of addition as commutative.

From understanding conservation of number lead on to counting on (seen and unseen).
eg Count **5** objects into a cloth bag. How many objects in the bag? Count **2** more objects into the bag. How many objects in the bag now?

5 + 2 = 7

Use a variety of methods to teach these concepts.
eg Jumping along a number track, using fingers, counting in unison, silent counting.

0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**

Use of number track and then numberline should be encouraged when appropriate to the child's needs.

<u>Number Bonds:</u>

At this stage it is also important to introduce number bonds to the children. They should get to know these by heart to 10 and 20. They should then be taught to use these facts to derive other number bonds such as to 100.

E.g. I know 3 + 7 = 10 → so I can work out that → 30 + 70 = 100

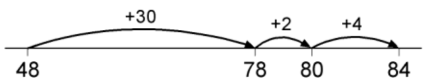
Stage 2: Numberlines and 100 squares:

Numberlines:

Numberlines can be used to support addition by counting on in steps of one. It is important that children understand that they can not count until there is a movement along the numberline.

Steps in addition can be recorded on a number line. The steps often bridge through a multiple of 10.

48 + 36 = 84



or

Children can use numberlines to record on. These can be given to them or they can write their own. Once children are secure with numberlines then they can use blank ones to record their own steps on.

Number lines can be used for the addition of two numbers right the way up through the school from 4 + 3 = 7 to 12.5 + 34.7 = 47.2 where the children are recording their own steps on them.

Using numberlines confidently for addition will also support the children with their subtraction method of counting on.

Hundred Squares:

Hundred squares can be used alongside number lines to support children's mental calculation skills they can also be used to reinforce their understanding of place value. Children should have experience of using these from 0-99 and 1-100. They should also work with squares that have missing numbers.

Stage 3 – Partitioning

Partitioning is the act of splitting up a number to make a calculation easier to manipulate. Children learn to partition from Year 1 and they can explore this using diennes, place value cards, straws or other objects to support their understanding and conservation of number. They should be encouraged to split the number up in a variety of ways including hundreds, tens and ones.

E.g. 43 can be partitioned into: 40 + 3 20 + 23 13 + 30 41 + 2 15 + 28

The next stage is to record mental methods using partitioning. Add the tens and then the ones to form partial sums and then add these partial sums together.

Record steps in addition using partitioning:

47 + 76 =

40 + 7 70 + 6

40 + 70 = 110 7 + 6 = 13

110 + 13 = 123

This exact recording method does not need to be stuck to – children tend to find a method of recording that works for them. However as children progress they should be pushed into presenting their partitioned calculations in a method that will support the introduction of the column method of addition.

Partitioned numbers are then written under one another: This method also links in with the subtraction method at this stage so children become familiar with the layout.

47 + 76 =

T		O
40	+	7
70	+	6

110 + 13 = 123

284 + 453

H		T		O
200	+	80	+	4
400	+	50	+	3

600 + 130 + 7 = 737

When working on this stage it is important to make sure you are **constantly reinforcing** the idea of hundreds tens and ones and that each number has a specific value. This will help children later on with their understanding of the compact vertical method for addition.

Partitioning can then be extended to include hundreds and thousands inline with children's progress and the objectives for their year group. This can really help support children's mental calculation skills. If addition of numbers cannot be completed mentally then the children should be moved to using the column method for addition.

Children should be taught to recognise what type of calculation a questions is asking of them. They should be able to look at an addition calculation and know if they can do it mentally or a written method is needed.

E.g.

Calculation	Mental	Written	Explanation
3456 + 1242 =	✓		This can be done mentally as the numbers that are to be added together do not cross any barriers. This calculation is just a simple case of adding the digits together and writing down the answer
2659 + 328 =	✓	✓	This could be done either way depending upon the confidence level of a child with manipulating numbers. The ones column when added crosses a barrier so the child could choose to use the column method for addition and carry or work mentally to find the answer
9567 + 4886		✓	This calculation is suited to the written method as all the columns cross a barrier and working mentally, whilst not impossible, could lead to mistakes being made.

<u>Stage 4 Column Method</u>

- In this method, recording is reduced further.
- Carry digits are recorded below the line, using the words 'carry ten' or 'carry one hundred', not 'carry one'.
- Write the initial letters of the place value columns above the columns until children become secure in their understanding. Do the same with decimal numbers.

E.g.

47 + 76 =	258 + 87 =	366 + 458 =
T O	H T O	H T O
47	258	366
+ 76	+ 87	+458
123	345	824
~~1~~~~1~~	~~1~~~~1~~	~~1~~~~1~~

This method is easily extended to include adding 4 digit numbers, decimals and numbers of different sizes.

Children, who cannot fully explain how this method works, **must go back to working on Part 1 of the column method**. This will help them to further concrete their understanding of the numbers that they are adding and the value that they hold.

Column addition remains efficient when used with larger whole numbers and decimals. Once learned, the method is quick and reliable.

Children must always be able to explain what they are doing in the quick method and be able to show that they know that if they have a 4 and a 3 in the hundreds column that it is not 4 + 3 but 400 + 300.

Subtraction

Many children have difficulties with calculations involving subtraction. They need to understand that the symbol - can be interpreted in a number of ways using a variety of language.

18 - 7   18 subtract 7

18 minus 7

What is the difference between 18 and 7?

What is the difference between 7 and 18?

How many more is 18 than 7?

How many less is 7 than 18?

18 take away 7

Decrease by 13…..

Fewer for concrete objects

Subtraction is sometimes 'taking away' and sometimes 'comparing' (finding the difference).
This needs to be reflected in the language used, and the word problems set.
It is important to model the concept of subtraction, practically.
Children need to develop a good understanding of place value and number facts.

Practical resources need to be used to model subtraction, both by the teacher and by pupils.

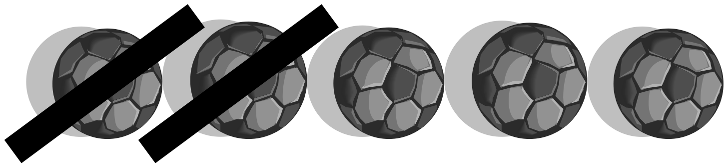
When presenting subtraction questions to the children they should always be presented horizontally even if they are using a vertical method for subtraction.

<u>Stage 1: Practical exploration of subtraction</u>

In practical activities and discussion begin to use the vocabulary involved in subtraction.

It should be noted that at this stage examples can be made of both types of subtraction sum (difference between numbers and taking away) Practical exploration of the concept Pictorial recording of the concept. . .teacher models recording method, child then uses teacher format to record as appropriate.

Eg Sarah has 5 balls, but 2 burst. How many are left?



"3 are left"

Pictorial recording alongside numbers (such as number cards; computer record etc.) Focus on language.

Eg Who has more/fewer apples than you?

8

3

Using pictorial representations develop into how many more/fewer?
e.g.
"He has 5 more apples than me". 3 + 5 = 8 to emphasise that 3 and 5 more is 8.
Use a numbered number line to count backwards in ones.

Number bonds:

Number bonds can be used to make the link between addition and subtraction.
E.g If children know 7 + 3 = 10 then they should be encouraged to realise that 10 – 7 = 3 or that 10 – 3 = 7. This knowledge once learnt can then be applied to more advanced calculations

Stage 2: Subtraction using a numberline

Steps in subtraction can be recorded on a number line.
The children should begin as they did with addition using a numbered numberline.
Once they are happy with this then they can move on to writing/drawing their own numberlines and recording their own jumps.

There are two methods for subtraction when using numberlines. You can start with the number that is being subtracted from and count back. Or you can start with the number that you are subtracting and count on to the other number in the question.

If counting on, it is essential that children understand why this method works. They must understand that they are counting up to find the difference between the value of the smaller number and the value of the larger number.

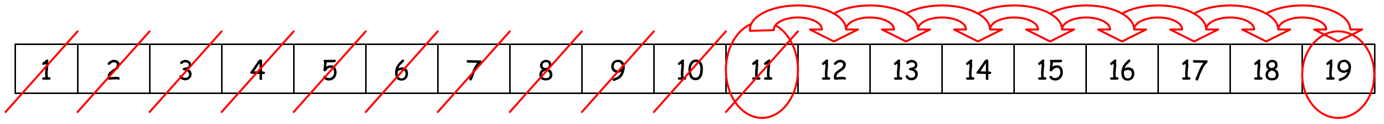
This can be show easily on a numberline by crossing out the value of the smaller number (number being subtracted).

e.g. 19 – 11 =

1. Identify the larger number
2. Cross out the numbers that have been "covered" by the smaller amount – finding the number 11 on the numberline and then scribbling out the numbers before it helps children to see why counting on works
3. Count up the rest of the numbers that have not been crossed out until they get to the target numbers

8 jumps of 1 unit = 8

So 19 – 11 = 8



This representation shows the cardinality of numbers. (9 on the number line is an ordinal number – 1-9 is the cardinal amount showing the number by using 2 hands helps children to understand the amount in relation to the number)

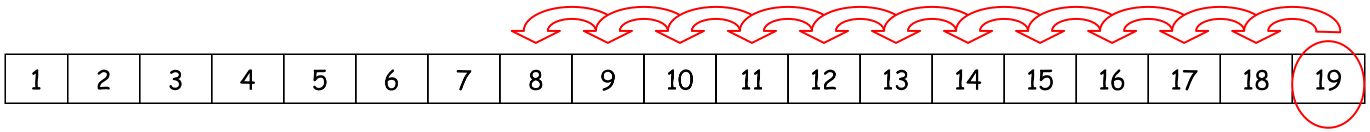
Children often won't use the difference in their work because they don't understand why it works – counting back makes sense because they are taking away – they might do it in a lesson however when given own choice they choose to count back

The other version is counting back. In this case you would find the number on a number line and then count backwards.

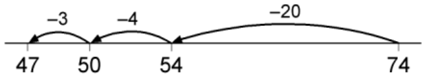
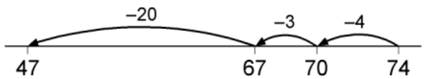
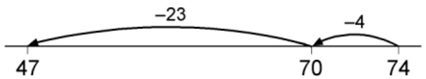
e.g. 19 – 11 =

1. Identify the larger number
2. Count backwards the amount that is being subtracted
3. The number you land on one the amount has been counted backwards is your answer

11 taken away / 11 subtracted



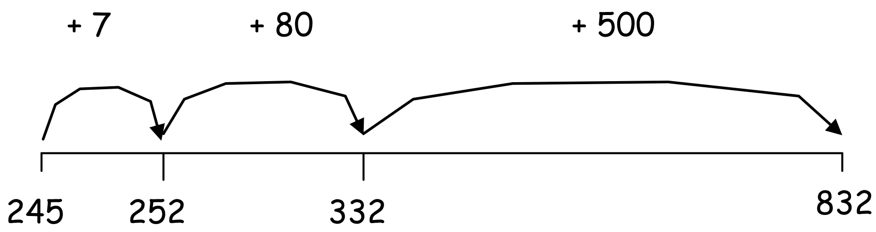
19 – 11 = 8

The steps can often bridge through a multiple of 10. e.g. 15 – 7 = 8  	74 – 27 = 47 worked by counting back: 
The steps may be recorded in a different order:  	or combined: 

There are a variety of ways that children will record their working out on numberlines – their work should always be legible and they should always be able to explain the reason why they chose to use that particular way to work out the answer

It is possible to go on to quicker methods of subtraction using a number line such as **digit matching** for counting on

e.g. 832 – 245 =



832 – 245 = 587

In the example above the digits are matched starting with the ones first. This method does require the children to be secure in their understanding of crossing boundaries. This method does however reduce the amount of jumps required to the size of the number (number of digits) and when it comes to adding the jumps up the answer can be almost read off the page with little further addition required.

<u>Stage 3: Partitioning</u>

Subtraction can be recorded using partitioning. This involves writing equivalent calculations that can be carried out mentally to find the result. When using partitioning it is important that the children understand that they only partition the number that they are taking away and that the partitioned numbers should be easier to take away than the initial number.

Eg. 74 – 27 =

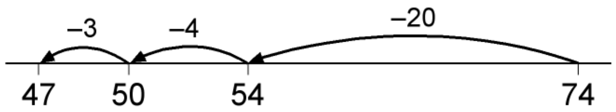
This involves partitioning the 27 into 20 and 7. These would then be subtracted from the 74 in turn.
Some children may need to partition the 74 into 70 + 4 or 60 + 14 to help them carry out the subtraction.

74 – 27 =

20 + 7

74 – 20 = 54 → 54 – 7 = 47 → 74 – 27 = 47   (or subtracting the ones first, then the tens)

This requires children to subtract a single-digit number or a multiple of 10 from a two-digit number mentally. The method of recording links to counting back on a number line or hundred square.



This is easily extended to 4 digit subtraction and subtraction using decimals.

The next step would be to introduce the column method for children. At this stage there are two methods available to the children to use

Method 1: Expanded decomposition

326 – 115 =

	H		T		O
	300	+	20	+	6
-	100	+	10	+	5
	200	+	10	+	1

= 211

(the numbers are then added together to make the answer)

326 – 115 = 211

This can then be extended to the expanded method with decomposition:

326 – 135 =

	200		120		
	~~300~~	+	~~20~~	+	6
-	100	+	30	+	5

100 + 90 + 1 = 191

(the numbers are then added together to make the answer)

326 -135 = 191

Stage 4: The Standard Compact Method - Decomposition

Builds on method 1 in the previous stage

It is important that children must know why this method works. They must be able to see the maths going on and be able to explain the fact that they are taking amounts from one column and exchanging them into others. If they can not do this then they must go back to the expanded version of decomposition

The numbers are no longer partitioned in the standard method.

E.g.

326 – 135 = 191

$$
\begin{array}{r}
{}^{2}\not{3}\,{}^{1}2\ 6 \\
-\ 1\ 3\ 5 \\
\hline
1\ 9\ 1 \\
\hline
\end{array}
$$

Multiplication

Written methods for multiplication

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. Children are entitled to be taught and to acquire secure mental methods of calculation and one efficient written method of calculation for multiplication which they know they can rely on when mental methods are not appropriate.

To multiply successfully, children need to be able to:

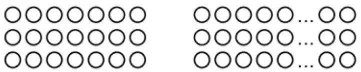
- add two or more single-digit numbers mentally;

- add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
- add combinations of whole numbers using the column method (see policy for addition).
- recall all multiplication facts to 10 × 10;
- partition number into multiples of one hundred, ten and one;
- work out products such as 70 × 5, 70 × 50, 700 × 5 or 700 × 50 using the related fact 7 × 5 and their knowledge of place value;

It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for multiplication.

The Distributive law for multiplication:

Children should be introduced to the principle of this law (not its name) in Years 2 and 3, for example when they use their knowledge of the 2, 5 and 10 times-tables to work out multiples of 7. Children should be taught this law as they progress to harder calculations
E.g.



$$7 \times 3 = (5 + 2) \times 3 = (5 \times 3) + (2 \times 3) = 15 + 6 = 21$$

An array is a useful tool for children to have and arrays should be taught alongside simple multiplication problems. Further up the school arrays can be used to understand area and perimeter of rectangles and L shapes

For all stages of multiplication the children must also learn about the link to division with the inverse

This is sometimes called the "family of four" where 2 multiplication operations are linked to 2 division ones (this also covers the commutative law of multiplication)
E.g.

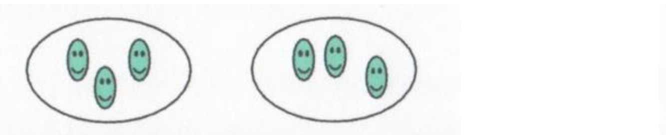
3 x 5 = 15 5 x 3 = 15 15 ÷ 5 = 3 15 ÷ 3 = 5

APPROXIMATIONS: When moving into TU x TU or HTU x TU then children should also be taught about the value of approximations. They should be able to approximate an answer by rounding and be able to state whether the actual answer will be higher or lower than the approximated one giving the reasons for that decision.

Stage 1: Practical exploration of multiplication

Children should be engaged in practical exploration of multiplication using everyday objects

Select two groups of objects to make a given total of objects (FS objective)



eg. Pairs of socks, gloves etc

Use real objects and pictorial representations to sort into equal sets, count in pairs.

Children must understand multiplication as doubling a number or repeated addition.

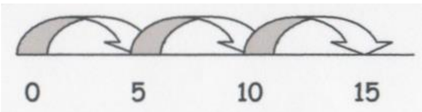
Stage 2: Multiplication as Repeated Addition

Children should understand that they can use repeated addition to work out answers to multiplication questions, and that this can be represented in numbers or as an array.

E.g. 2 x 5 = 2+2+2+2+2 or  or 

It must be clear to children that there are different ways that the array can be represented
This can also be shown in the presentation of the multiplication question E.g. 2 x 5 or 5 x 2

Children can also use an empty numberline to represent multiplication as repeated addition



5+5+5=15 and 3 sets of 5 = l5

Stage 3: Multiplication through Partitioning

Children can use the partitioning and recombining method to work out answers to multiplication questions. Here numbers are split so that calculations can be linked to the tables knowledge of the children enabling them to work out the answers at a quicker pace supporting mental calculation.

TO x O

36 x 5 = (30 + 6) x 5

In this method of recording please refer to the + as *"and"* with the children

30 x 5 = 150 6 x 5 = 30

150 + 30 = 180

HTO x O

235 x 7 = (200 + 30 + 5) x 7

200 x 7 = 1400 30 x 7 = 210 5 x 7 = 35

1400 + 210 + 35 = 1645

TO x TO

23 x 57 = (20 + 3) x (50 + 7)

20 x 50 = 1000 20 x 7 = 140 3 x 50 = 150 3 x 7 = 21

1000 + 140 + 150 + 21 = 1311

With any of the above methods for partitioning the children are able to set out their work in a variety of was and as with addition as long as they are able to confidently explain their reason for setting out their work then this is fine

Once children are secure with partitioning numbers then they can be introduced to the grid method for multiplication. This can be used to help organise their writing and avoid missing out calculations (which can happen when partitioning).

The use of arrays can be a good way of explaining why the grid method works.

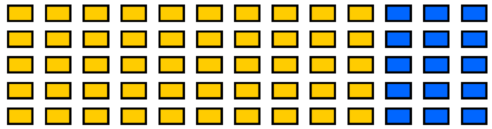
E.g.

13 x 5 If we compare a grid with an array we can see how the grid is just splitting up the array in to more manageable chunks

x	10	3
5	50	15

10 3

5



The array makes it possible for children to actually count up the parts to work out the answer – this can then be linked to repeated addition and then back to multiplication. Children could also use unifix cubes to create visual representations that can help them to have a more relational understanding of maths.

The method can also be used to support those who often make a mistake with partitioning as it provides a scaffold for the children in terms of partitioning

Using the grid method links to coordinates (shape and space) and Carroll diagrams (data handling).

TO x O

25 x 7

x	7
20	140

or

x	20	5
7	140	35

5	35

140 + 35 = 175

140 + 35 = 175

The answers to the questions should be written within the grid. These numbers are then added together either mentally or using a written addition method that the children are familiar with

HTO x O — Children who are familiar with the grid method for TO x O are often able to pick up the grid method for HTO x O quite simply

342 x 4

x	4
300	1200
40	160
2	8

Or

x	300	40	2
4	1200	160	8

1200 + 160 + 8 = 1368

1200 + 160 + 8 = 1368

TO x TO

24 x 43

X	40	3
20	800	60
4	160	12

800 + 60 = 860
160 + 12 = 172

```
   8 6 0
 + 1 7 2
 -------
 1 0 3 2
   1
```

<u>THE GRID METHOD REILES ON THE FACT THAT THE CHILD CAN SECURELY AND CONFIDENTLY ADD</u>

Stage 4: Compact Method

This method uses the children's knowledge of their tables to support large calculations. Children must be confident and able to explain the value of each digit in their calculations.

38 x 7

$$\begin{array}{r} 38 \\ \times \quad 7 \\ \hline 266 \\ \hline {}_{5} \end{array}$$

Here the larger number (top) is multiplied by the smaller number (bottom), starting with the ones first. So 7 x 8 = 56 – The 6 is put in place in the answer line however the 5 is carried underneath as it will add on to the total of the tens column multiplication. So 7 x 3 is 21 + 5 is 26. The 6 can now be put into place and the 2 can also move in as there is nothing in the hundreds column to calculate.

Stage 4 can then be extended easily to include HTOx O, ThHTO x O and beyond. Children who are secure in their ability to multiply TO x O in the same method have little problems adjusting the calculations to include larger amounts.

Stage 5: Long Multiplication

This method again uses multiplication fact knowledge to help solve larger calculations where number with more than 1 digit is being multiplied by a 2 digit number. Ensuring that the digits are put into the correct columns is paramount to enable the correct answer to be calculated at the end.

E.g. 45 x 23

		4	5	
	x	2	3	
	1	3	5	← The answer here is for 45 x 3 (as in the compact method)
		1		
	9	0	0	← This is the answer for 20 x 45 – a 0 is put in the ones column due to the fact that you are multiplying by a ten and not a unit
	1			
1	0	3	5	← The two answers are then added together (as in partitioning)

This method can then be adapted for more complex calculations:

		4	2	8	7
	x			4	6
	2	5	7	2	2
		1	5	4	
1	7	1	4	8	0
	1	3	2		
1	9	7	2	0	2
		1	1		

An expanded version can be used to support the children in their understanding as a half way point to using long multiplication

38 x 7 =

$$\begin{array}{r} 30+8 \\ \times \underline{\quad 7} \\ 210 \\ \underline{56} \\ \underline{266} \end{array} \qquad \begin{array}{l} 30\times 7 = 210 \\ 8\times 7 = \ \ 56 \end{array} \qquad \begin{array}{r} 38 \\ \times \underline{\quad 7} \\ 210 \\ \underline{56} \\ \underline{266} \end{array}$$

All multiplication stages rely on children having two things:

- **an excellent tables knowledge** and **rapid recall of the tables facts**.
- **A secure understanding of column addition**

If they do not have these things then it can be hard for the children to see the compact methods as more efficient ways of recording their calculations and they will struggle to achieve when using these methods.

Division

To divide successfully in their heads, children need to be able to:

- understand and use the vocabulary of division – for example in 18 ÷ 3 = 6, the 18 is the dividend, the 3 is the divisor and the 6 is the quotient;
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways;
- recall multiplication and division facts to 10 × 10, recognise multiples of one-digit numbers and divide multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value;
- know how to find a remainder working mentally – for example, find the remainder when 48 is divided by 5;
- understand and use multiplication and division as inverse operations.

It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for division.

To carry out written methods of division successful, children also need to be able to:

- understand division as repeated subtraction;
- estimate how many times one number divides into another – for example, how many sixes there are in 47, or how many 23s there are in 92;
- multiply a two-digit number by a single-digit number mentally;
- subtract numbers using the column method.

Stage 1: Practical exploration of division

Children need to be able to compare 2 or 3 sets of objects using the language 'more', 'less' and the same.

eg.

"There are more bears than cars or footballs"
"There are less footballs than bears"
"There is the same amount of cars and footballs"

Children also need experience of sharing objects into 2 equal groups and counting how many in each group. (discuss what happens when there is one left over)

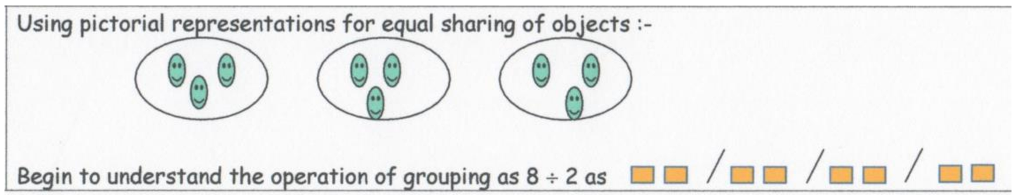
There are 7 dolls that can be split into 2 groups of 3 with 1 left over.

The 1 left over is a remainder as it can't go into both groups or they would be different in size and when we share into groups we share equally.

Children could use practical resources such as pencils/sweets/grapes to use to share and should be drawing pictures to explain their working.

Eg. Millie had 6 toffees, she gave half to her friend. How many toffees did she have?

This can be extended to more than 2 groups / sets of objects:



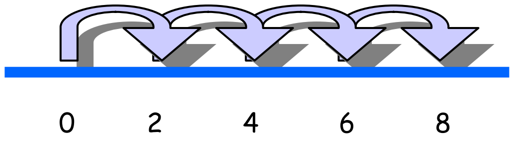
Stage 2: Use grouping to solve division problems and begin to record on number lines – supporting mental calculation

Children must be secure with their ability to group numbers and divide before starting to record on their numberlines.

Using arrays and unifix cubes to help represent their maths can be used to help them link between the hands on (practical representations) and the abstract (written).

Eg.

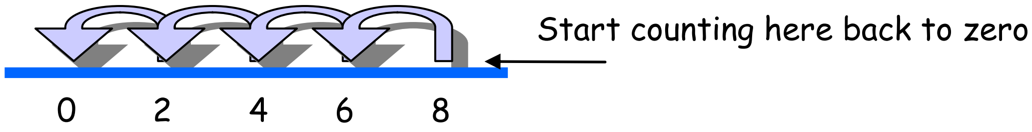
Grouping – There are 8 sweets. How many people can have 2 each?
(How many 2s make 8?)



There are 4 jumps so 4 people can have 2 sweets each.

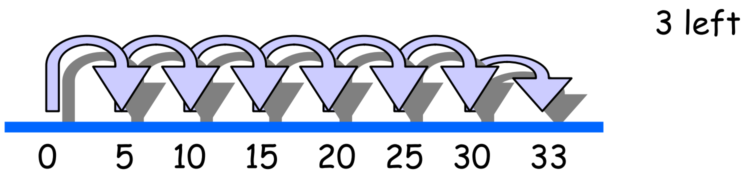
You can also work backwards to zero.

Eg.   There are 8 sweets. How many people can have 2 each?



This can be extended to division with remainders:

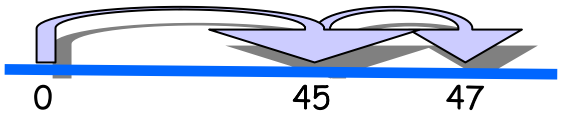
Eg. 33 ÷ 5



6 jumps with 3 remainder -   Answer = 6 remainder 3

Using an empty numberline gives children the ability to select their own jumps. Children must be secure in the numbers that they are working within to make this a success. They must also be secure with their tables.

Eg. 47 ÷ 5



9 x 5 (9 lots of 5)

2 left over (remainder 2)

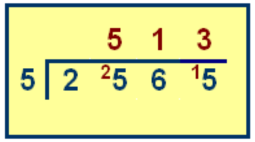
Answer = 9 remainder 2

Stage 3: Compact method for division

Children can then use a more efficient method for division

Eg.

2565 ÷ 5



Thought process:

- The 5 into 2 (2 thousands) won't go so you rewrite the 2 in the hundreds column making 25 (25 hundreds = 2500)
- 5 into 25 (25 hundreds) goes 5 times exactly this number is written above the line in the hundreds column
- 5 into 6 (6 tens) goes once with 1 (1 ten) remainder. The 1 goes above the line in the tens column to show that the 5 went in once and
  the 1 (1 ten) remainder is written in the ones
- 5 into 15 (15 ones) goes 3 times exactly.

So 2565 ÷ 5 = 513

Where there is a remainder the children can do one of 2 things.

1. They can either write the remainder as a remainder or a fraction
2. They can add in a decimal place and a 0 and carry on writing the part that is being transferred into the next column

Eg.

346 ÷ 4 =

$$\begin{array}{r|l} & 8\ \ 6\ .\ 5 \\ \hline 4 & 3\ {}^{3}4\ {}^{2}6\ .\ {}^{2}0 \end{array}$$

346 ÷ 4 = 86.5

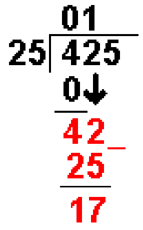
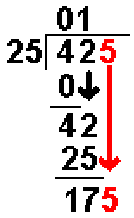
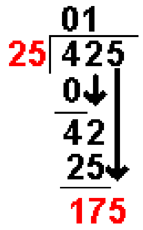
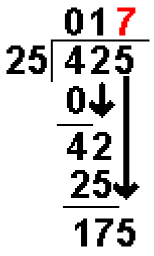
When setting problems like this children must be aware that some division problems may not have an end as the number has a repeating pattern in the decimals. Children can be made aware of this so that when they find the repeated sequence they stop working out the problem. They could however be instructed to round to either 1 or 2 decimal places instead.

Stage 4: Long Division

Long division is used when a number is divided by a number with more than one digit. Explained Below.

25 | 425	4 ÷ 25 = 0 remainder 4	The first digit of the dividend (4) is divided by the divisor.
0 25 | 425		The whole number result is placed at the top. Any remainders are ignored at this point.

0 25 | 425 0	25 × 0 = 0	The answer from the first operation is **multiplied** by the divisor. The result is placed under the number divided into.
0 25 | 425 0 4	4 - 0 = 4	Now we **subtract** the bottom number from the top number.
0 25 | 425 0↓ 42		Bring down the next digit of the dividend.
0 25 | 425 0↓ 42	42 ÷ 25 = 1 remainder 17	**Divide** this number by the divisor.
01 25 | 425 0↓ 42		The whole number result is placed at the top. Any remainders are ignored at this point.
01 25 | 425 0↓ 42 25	25 × 1 = 25	The answer from the above operation is **multiplied** by the divisor. The result is placed under the last number divided into.

	42 – 25 = 17	Now we **subtract** the bottom number from the top number.
		Bring down the next digit of the dividend.
	175 ÷ 25 = 7 remainder 0	**Divide** this number by the divisor.
		The whole number result is placed at the top. Any remainders are ignored at this point.

017 25 ⟌ 425 0↓ 42 25↓ 175 175	25 × 7 = 175	The answer from the above operation is **multiplied** by the divisor. The result is placed under the number divided into.
017 25 ⟌ 425 0↓ 42 25↓ 175 − 175 000	175 – 175 = 0	Now we **subtract** the bottom number from the top number.
		There are no more digits to bring down. The answer must be 17

<u>Other Possible Method: Chunking (also known as repeated subtraction using multiplication facts)</u>

Chunking uses the children's knowledge of their tables combined with their ability to be able to vertically subtract to work out the answer to division calculations.

If children are not secure in these things then they will struggle to achieve using the chunking method

TO ÷ O with no remainder

E.g. 95 ÷ 5 = (How many lots of 5 go into 95?)

```
5 | 95
    50   (10 x 5)   We know 10 lots of 5 go in so we take those away
    45
```

$\underline{45}$ (9 x 5) We know we can get another 5 lots of 5 out and take those away

0 There is nothing left so we stop and add up how many lots we have used 10 + 9 = 19

95 ÷ 5 = 19

This can then be extended to HTU ÷ U

HTO ÷ O or TO ÷ O with a remainder

247 ÷ 5 =

```
5 | 247
    100     (20 x 5)   If 10 x 5 is 50 then we know 20 x 5 is 100 and we take that many away
    147
    100     (20 x 5)   We can take another 20 lots of 5 away
     47
     45     (9 x 5)    We know from our tables that 9 x 5 is 45 and 45 can be taken away from 47
      2                We can't get another lot of 5 out so we stop and 3 is the remainder
```

We have now stopped so we count up how many lots of 5 we have used

Answer = 49 remainder 2

The answer can also be written as $49\frac{2}{5}$ (because there are 2 parts left out of a possible 5)

You could also convert $\frac{2}{5}$ into a decimal giving the answer 49.4

Children can then use the 'chunking' method to be able to calculate HTU ÷ TU with remainders, moving towards a more efficient method with fewer steps.

Solve 977 ÷ 36

```
36 | 977
     360   (10 x 36)
     617
     360   (10 x 36)
     257
     180   (5 x 36)
      77
      72   (2 x 36)
       5
```

moving towards →

```
36 | 977
     720   (20 x 36)
     257
     180   (5 x 36)
      77
      72   (2 x 36)
       5
```

Answer: 27 $\frac{5}{36}$

Decimals and Money:

All of the examples in the policy do not include specific examples of decimals or money as the same method can be used with just a slight tweak to the layout

Addition	Subtraction	Multiplication	Division
In partitioning the decimal would be partitioned just like any other amount. With relation to money the value of the number would either come before or after	As with addition the decimal would be partitioned just like any other number. In relation to money the value of the number would also be the same coming before or after Eg. £3·40 or 340p	As with addition the decimal would be partitioned just like any other number. In relation to money the value of the number would also be the same coming before or after Eg. £3·40 or 340p	For decimals in division children can just write them into their layout or on the numberline With money:

<table>
<tr>
<td>Eg. £3·02 or 302p

Column method:

£3·40 + £2·30 =

  £ 3·40
+ £ 2·30
  £ 5·70</td>
<td>Same layout as addition for the compact method

Expanded method

£3·50 - £1·75 =

  £3·00 + £0·50
- £1·00 + £0·75

  £2·00 + -£0·25 = £1·75</td>
<td>With the grid method:

£23 x £45
<table><tr><td>x</td><td>£20</td><td>£3</td><td></td></tr><tr><td>£40</td><td>£800</td><td>£120</td><td>£ 920</td></tr><tr><td>£5</td><td>£100</td><td>£15</td><td>£ 115</td></tr><tr><td></td><td></td><td></td><td>£1035</td></tr></table>
Vertical Method:

  £ 3·45
x   3
  £10·35
    1 1</td>
<td>£4·68 ÷ 4 =

    £1 · 1 7
4 | £4 · 6 ^{2}8</td>
</tr>
</table>

The above are only suggested examples that could be followed. As with partitioning the children are allowed to place the £ or p where ever they want, as long as they can explain why they have put it there and it makes sense mathematically.

Appendix 1: Overview of the calculations and the year group in which they are introduced

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Addition	Solve problems through practical methods use of numberlines/100 squares etc.	Partition numbers to aid addition – start to arrange in columns	Introduce column addition with numbers up to 3 digits and numbers with 1 decimal place	Column method - 4 digits – and numbers with up to 2 decimal places	Column method more than 4 digits	Multi digit calculations
Subtraction	Solve problems through practical methods use of number lines/100	Partition to aid subtraction – use of numberlines and 100 squares	Introduce column subtraction (decomposition) up to 3 digits – and	Column method 4 digits– and numbers with up to 2 decimal places	Column method more than 4 digits	Multi digit calculations

	squares etc.		numbers with up to 2 decimal places			
Multiplication	Solve problems through practical methods and knowledge of tables including using the signs x and = correctly	Use arrays / repeated addition / multiplication facts / practical resources including using the signs x and = correctly	Efficient written method -2 digit by 1 digit column method $\begin{array}{r} 38 \\ \times\ \ 7 \\ \hline 266 \\ _{5} \end{array}$	Efficient methods 2 and 3 digit numbers by 1 digit number	4 digit numbers by 1 and 2 digits (long multiplication for 2 digits) Introduce long multiplication	Multiply multi-digit numbers up to 4 digits by a 2 digit whole number using long multiplication
Division	Solve problems through practical methods including using the signs = and ÷ correctly	Use arrays / repeated addition and subtraction / division facts / practical resources including using the signs = and ÷ correctly	Introduce the efficient method $\begin{array}{r} 1\ 2 \\ 4 \overline{) 4\ 8} \end{array}$ No remainders	Secure the efficient method – no remainders (Can include decimals)	Divide 4 digit by 1 digit numbers and interpret remainders appropriately for context (fractions or whole number remainders) Money as well	Divide up to 4 digits by a two digit whole number using long division

Appendix 2: Numberlines and 100 squares/number grids

Numberlines are an invaluable resource for children to have access to

Numberlines can be:

- Blank
- labelled in different amounts (1's – 2's – 5's etc)
- made of fractions
- go into negative numbers
- bent to look like a clock (time)
- arranged in grids

Grids are often overlooked as numberlines. Some children do not make the link between numberlines and 100 square grids. A useful activity can be to cut up a 100 square grid and then turn it into a number line.

Using grids of different sizes can also encourage children to look at numbers in a different way.

E.g.

- using a 0 – 99 number grid
- using a grid where the numbers are in the reverse of the normally associated way
- using a grid where the last number in the line is a multiple of 4,5,6 etc.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16
17	18	19	20
21	22	23	24
25	26	27	28
29	30	31	32

Also having missing number numberlines and grids can be a way of reinforcing rules of number

94		
97		99
	101	

Appendix 3: Grading of Difficulty for questions

Below is a grid that lays out an order in which questions can be given to children looking at the difficulty of working out each operation.

Addition	Subtraction	Multiplication	Division
1.No carrying e.g. 23 315 + 42 + 624	1. No adjustment e.g. 47 864 - 23 - 621	1. No carrying e.g. 32 44 x 3 x 2	1. Single digit division, no remainder, no carrying e.g. 69 ÷ 3 264 ÷ 2
2. Extra digit in answer e.g. 94 561 +73 + 718	2. Adjustment T to ones e.g. 52 432 - 36 -217	2. Extra digit in answer e.g. 32 51 x 4 x 4	2. Remainder, no carrying e.g. 68 ÷ 3
3. Carrying ones to T (tens) e.g. 47 237 +25 +516	3. Adjustment H to T e.g. 437 618 -182 -217	3. Carrying but keeping in same decade e.g. 83 34 x 4 x 7	3. No remainder, carrying e.g. 45 ÷ 3
4. Carrying T to H e.g. 371 293 +485 +541	4. Adjustment H to T and T to ones 432 - 187	4. Carrying and going into next decade e.g. 78 68 x 7 x 8	4. Remainder, carrying e.g. 47 ÷ 3
5. Carrying ones to T and T to H e.g. 376 295 +485 +547	5. Noughts e.g. 470 700 604 - 142 - 236 - 347	5. Noughts e.g. 202 430 x 4 x 6	5. Placing of the quotient e.g. 287 ÷ 7
6. More than two numbers to be added e.g. 463 921 +759		6. Multiplying by multiples of 10 87 416 x 10 x 60	6. Noughts in quotient e.g. 816 ÷ 4 5608 ÷ 8
7. Different numbers of digits e.g. 23 4756 375 20375 + 48 + 752		7. Long multiplication e.g. 47 832 x 23 x 74	**Two-digit division** 7. No remainder e.g. 64 ÷ 32 93 ÷ 31
			8. Similar but remainder e.g. 29 ÷ 13 97 ÷ 31
			9. Quotient not so apparent e.g. 56 ÷ 22 92 ÷ 41
			10. Placing the quotient e.g. 126 ÷ 21 224 ÷ 32
			11. No remainder e.g. 483 ÷ 21 736 ÷ 32
			12. Remainder e.g. 718 ÷ 33
			13. Noughts in quotient e.g. 6834 ÷ 17
			14. Divisors like 29, 39, 48
			15. Divisors like 45, 37, 24, 56